Critical Evaluation of the Flow Rate-Pressure Drop Relation Assumed in Permeability Models

K. M. NG and A. C. PAYATAKES

Chemical Engineering Department
University of Houston
Houston, TX 77004

There are two types of porous medium models for predicting permeability: network and statistical models. Networks of cylindrical tubes were proposed by Fatt (1956), Probine (1958), Josseling de Jong (1958), Ksenzhek (1963), Nicholson and Petropoulous (1971), Simon and Kelsey (1971, 1972), and Dullien (1975), whereas Payatakes et al. (1980) developed model networks in which the nodes are connected by constricted unit cells, rather than cylindrical tubes. In statistical models, as proposed by several workers (Saffman, 1960; Haring and Greenkorn, 1970; Guin et al., 1971; Payatakes and Neira, 1977), the flow channels are also randomly sized and oriented but not interconnected in any prescribed fashion.

For each unit cell of known geometry and orientation, there are two unknowns, namely, the flow rate through the cell q_{uc} and the pressure drop along its length ΔP_{uc}. It is customary in statistical models to assume that ΔP_{uc} is equal to the value obtained by assuming that the porous medium is a structureless continuum even at the microscopic scale, that is

$$\Delta P_{uc} = -h_{uc} \cdot \nabla P = -|\nabla P| h_{uc} \cos\alpha \tag{1}$$

The advantage of this assumption is its simplicity. The disadvantage is that the effect of interaction among unit cells is omitted. Furthermore, even though continuity is satisfied at the macroscopic level, in the case of networks it is violated at the microscopic level, since the flow rates through the unit cells connected to a node usually do not add up to nil. In this work, the validity of Eq. 1 is examined. To fix ideas, we perform the analysis using the network model proposed by Payatakes et al. (1980).

NETWORK ANALYSIS OF SINGLE PHASE FLOW

Consider a finite, three-dimensional cubic network of constricted unit cells, which will be called a unit block. The size of a unit block is indicated by the number of constricted unit cells along each coordinate axis, $N_x \times N_y \times N_z$; there are $N_x N_y (3N_z - 2)$ unit cells in the block. Under creeping flow conditions, the flow rate through each unit cell is given by Payatakes and Neira (1977) as

$$q_{uc} = \frac{\pi c_2 d^3}{4\mu(-\Delta P_1^*)} \Delta P_{uc} \tag{2}$$

To perform the analysis, random conductance values are first assigned to the unit cells based on the constriction size distribution (Ng and Payatakes, 1980). Then a pressure difference is applied across the unit block in the z direction. To eliminate the edge effect, identical replicas of the unit block are placed on the other four surfaces, thus forming an infinite slab. Using the node analysis in electrical circuit theory (Dosoer and Huh, 1969), we can readily obtain a set of equations for v, the node pressure vector, which represents the set of pressure values at the nodes of the network.

$$Gv = J \tag{3}$$

The nodal conductance matrix G is symmetric and sparse. The equivalent flow rate vector J represents the applied pressure difference. Equation 3 is solved by Gaussian elimination. Using Darcy's law, the permeability is given by

$$k = \frac{Q\mu}{A|\nabla P|} \tag{4}$$

Sufficiently many computational realizations are made on the unit block to ensure that the results become statistically stationary.

It should be noted that the pressure gradient is chosen to be along the z direction for convenience only. The same permeability will be obtained with the pressure gradient oriented at any arbitrary angle relative to the unit cell network, since the cubic network is isotropic for single phase flow.

RESULTS AND DISCUSSION

The model is applied to a sphere pack and a 100 × 200 sandpack (Payatakes et al., 1980). The mean permeability is compared with the experimental value and the theoretical value based on the simplifying assumption in Table 1. The network permeability (based on Eqs. 2 and 3) is about 25% lower than that based on the statistical model (Eqs. 1 and 2). Figure 1 shows why the permeability is inflated. Based on Eqs. 1 and 2, the flow rate increases

Table 1. Comparison of the Experimental Permeability Value with the Theoretical Values of the Network and Statistical Models

Packing	Permeability, mm²		
	Experiment	Network Model	Statistical Model
Glass bead	2.34×10^{-4}	3.01×10^{-4}	3.97×10^{-4}
100 × 200 sand	3.55×10^{-6}	1.83×10^{-6}	2.46×10^{-6}

K. M. Ng is currently at the Department of Chemical Engineering, University of Massachusetts, Amherst, MA 01003.

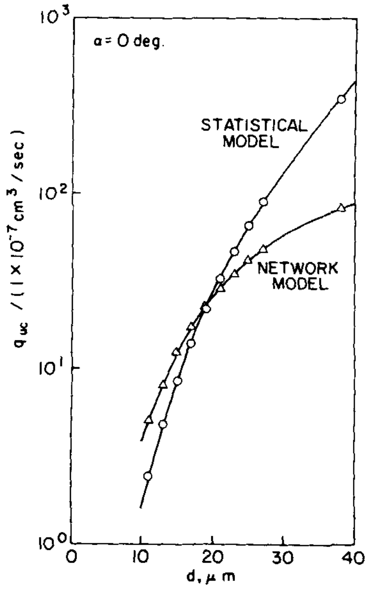

Figure 1. The volumetric flow rate in a unit cell is plotted against its constriction size for both statistical and network models.

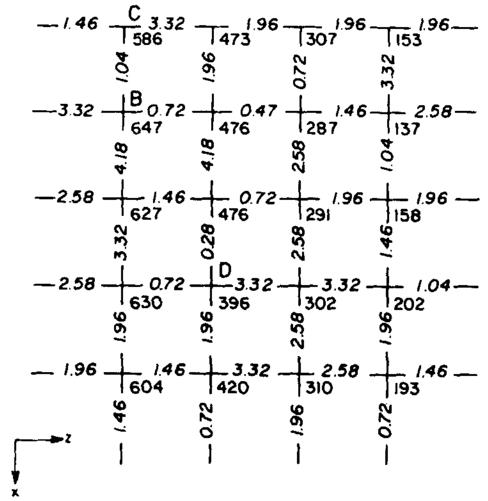

Figure 3. A layer of a unit block on which node pressures and conductances of each branch are displayed.

rapidly with the size of the unit cell. In reality, the flow rate in a unit cell is greatly influenced by its neighbors and increases slower than the prediction of Eqs. 1 and 2. The network model data for the sandpack which are obtained by sampling, with appropriate averaging, 4800 unit cells lying in the direction of the pressure gradient, show that the flow rate is greatly reduced in large unit cells, whereas it is increased in the smaller unit cells. The pressure gradient used is 1.5×10^4 dyne/cm^2/cm. A more dramatic fact is revealed by plotting the pressure drop along unit cells vs. unit cell size. The pressure drop actually decreases with increasing size, whereas Eq. 1 presumes exactly the opposite trend (Figure 2).

To illustrate the actual situation, a layer of the three-dimensional network is shown in Figure 3. The number along each branch is the corresponding conductance, $G/(1 \times 10^{-8}$ cm^4 s/g); the number at each node is the calculated node pressure (dyne/cm^2). Consider point B. When the conductance immediately downstream (0.72) is small compared to the ones adjacent to it, the fluid is forced to flow to those larger unit cells. Consequently, the node pressure at B is higher than that at C and the flow rate in the smaller unit cell (with conductance 0.72) is raised. Now consider point D. Even though there is a large cell (conductance 3.32) downstream, the unit cell upstream is so small that it does not provide enough flow volume to it. To some extent, the difference is reduced by flows from the side branches, but this means that the fluid has to travel a longer path and the flow rate is lower than what Eq. 1 would give.

In conclusion, the present results show that the simplifying assumption represented by Eq. 1 can deviate quite substantially from reality. Actually, the pressure drop along a unit cell is inversely proportional to its conductance, rather than being proportional to its projected length on the direction of the pressure gradient. These observations underline the significant yet often overlooked fact that transport phenomena in porous media are strongly affected by the cooperative behavior of a large ensemble of pores (unit cells). Only network analysis can take in account this aspect of transport in permeable media.

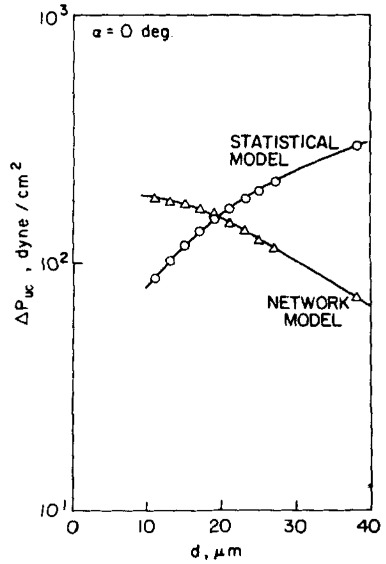

Figure 2. The pressure drop along a unit cell is plotted against its constriction size for both statistical and network models.

NOTATION

A	= cross-sectional area of a unit block
c_2	= constant (see Payatakes et al., 1980)
d	= minimum diameter of a unit cell
G	= node conductance matrix
h_{uc}	= axial length of an extended unit cell
J	= equivalent flow rate vector
k	= absolute permeability
N_x, N_y, N_z	= number of unit cells along each coordinate axis
P	= pressure
Q	= total flow rate across A
q_{uc}	= flow rate through a unit cell
v	= node pressure vector
x, z	= coordinates

Greek Letters

α	= angle between the axis of a unit cell and the macroscopic flow direction

ΔP_{uc}	= pressure drop along an extended unit cell
ΔP_1^*	= dimensionless pressure drop along an extended unit cell at $N_{Re} = 1$, neglecting inertial effects
∇P	= macroscopic pressure gradient
μ	= dynamic viscosity

LITERATURE CITED

Dosoer, C. A., and E. S. Kuh, *Basic Circuit Theory*, McGraw-Hill, New York (1969).

Dullien, F. A. L., "New Network Permeability Model of Porous Media," *AIChE J.*, **21**, 299 (1975).

Fatt, I., "The Network Model of Porous Media," (parts 1 to 3), *Petrol. Trans. AIME*, **207**, 144 (1956).

Guin, J. A., D. P. Kessler, and R. A. Greenkorn, "The Permeability Tensor for Anisotropic Nonuniform Porous Media," *Chem. Eng. Sci.*, **26**, 1475 (1971).

Haring, R. E., and R. A. Greenkorn, "A Statistical Model of a Porous Medium with Nonuniform Pores," *AIChE J.*, **16**, 477 (1970).

Josselin de Jong, G., "Longitudinal and Transverse Diffusion in Granular Deposits," *Trans. Am. Geophys. Union*, **39**, 67 (1958).

Ksenzhek, O. S., "Capillary Equilibrium in Porous Media with Intersecting Pores," *Russ. J. Phys. Chem.*, **37**, 691 (1963).

Ng, K. M., and A. C. Payatakes, "Stochastic Simulation of the Motion, Breakup and Stranding of Oil Ganglia in Water-Wet Granular Porous Media During Immiscible Displacement," *AIChE J.*, **26**, 419 (1980).

Nicholson, D., and J. H. Petropoulous "Capillary Models for Porous Media. III. Two-Phase Flow in a Three-Dimensional Network with Gaussian Radius Distribution," *J. Phys. D. Appl. Phys.*, **4**, 181 (1971).

Payatakes, A. C., and M. A. Neira, "Model of the Constricted Unit Cell Type for Isotropic Granular Porous Media," *AIChE J.*, **23**, 922 (1977).

Payatakes, A. C., K. M. Ng, and R. W. Flumerfelt, "Oil Ganglia Dynamics During Immiscible Displacement. Model Formulation," *AIChE J.*, **26**, 430 (1980).

Probine, M. C., "An Electrical Analogue Method of Predicting the Permeability of Unsaturated Porous Materials," *Brit. J. Appl. Phys.*, **9**, 144 (1958).

Saffman, P. G., "Dispersion Due to Molecular Diffusion and Microscopic Mixing in Flow Through a Network of Capillaries," *J. Fluid Mech.*, **7**, 194 (1960).

Simon, R., and F. J. Kelsey, "The Use of Capillary Tube Networks in Reservoir Performance Studies. I. Equal-Viscosity Miscible Displacements," *SPE J.*, **11**, 99 (1971).

———, "The Use of Capillary Tube Networks in Reservoir Performance Studies. II. Effect of Heterogeneity and Mobility on Miscible Displacement Efficiency," *SPE J.*, **12**, 345 (1972).

Manuscript received Aug. 5, 1981; revision received Apr. 10, 1984, and accepted Apr. 30.